Negative Pressures and Energies in Magnetized and Casimir Vacua

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We study the electron-positron vacuum in a strong magnetic field B in parallel with Casimir effect. Use is made of the energy-momentum tensor, taken as the zero temperature and zero density limit of the relativistic quantum statistical tensor. In both magnetic field and Casimir cases it is shown the arising of anisotropic pressures. In the first case the pressure transversal to the field B is negative, whereas along B an usual positive pressure arises. Similarly, in addition to the usual negative Casimir pressure perpendicular to the plates, the existence of a positive pressure along the plates is predicted. The anisotropic pressures suggests a flow of the virtual particles in both cases. By assuming regions of the universe having random orientation of the lines of force, cosmological consequences are discussed in the magnetic field case.

INTRODUCTION

There is some similarity among the effects of an external field and certain boundary conditions. For instance, this is the case of virtual electron-positron pairs of vacuum under the action of an external constant magnetic field, and the virtual photon field modes inside the cavity formed by two parallel metallic plates (Casimir effect [1],[2]. For more recent developments see [3]). Both problems bear some interesting analogy, the metallic plates keeping bounded the motion of virtual photons perpendicular to the plates whereas the external field bound electrons and positrons to Landau quantum states in the plane perpendicular to it.

In both cases, by starting from the expression for the zero point energy density of vacuum, and after subtracting some divergent terms, a finite negative energy density is left. From it, one can obtain the corresponding energy-momentum tensor density (in what follows, when referring to the energy-momentum tensor, we would mean its density). Its spatial components contain both negative and positive pressures: in the magnetic field case, an axially symmetric transverse negative pressure, and a positive pressure along the magnetic field. In the Casimir case, in addition to the well known negative pressure, it is found an axially symmetric positive pressure inside the cavity and along the plates.

The comparative study of these two QED vacua (in an independent way, but following a common method) is interesting in itself, exhibiting the property of anisotropic pressures, having negative values in some directions, as well as negative energy density.

We assume the magnetic field characterized by the microscopic magnetic field pseudo-vector $B_i = \epsilon_{ijk}\mathcal{F}_{jk}$ (where $\mathcal{F}_{jk}$ is the spatial part of the electromagnetic field tensor $\mathcal{F}_{\mu\nu}$), leading to the breaking of the spatial symmetry: the spinor wavefunctions and spectrum of charged particles having axial symmetry, their motion being bounded perpendicularly to B [4]. Its characteristic length is $\lambda_L(B) = \sqrt{\hbar c/2eB}$, and this is valid also for the zero point modes of vacuum. In the case of the the well-known Casimir effect it is produced when two parallel metallic plates are placed in vacuum, leading to the vanishing of the electric field component tangential to the plates. The plates having characteristic diameter a and separated by a distance d, with $a >> d$ (we shall assume in what follows $a \to \infty$). Only modes whose wave vector components perpendicular to the cavity are integer multiples of $k_{03} = \pi/d$, are allowed inside it. This makes the zero point electromagnetic modes inside the box axially symmetric in momentum space. In both problems there is a quantity characterizing the symmetry breaking, (and the extension of the wave functions in some direction). These quantities are respectively, the pseudo-vector B_i, determining $\lambda_L(B)$, and the basic vector momentum $\mathcal{P}_i = \mathcal{P}\delta_{3i}$, perpendicular to the plates (which are taken parallel to the x_1, x_2 plane). Here $\mathcal{P} = \hbar k_{03}$ and the length d characterizes the extension of the wave function perpendicular to the plates.

In the magnetic field case [4], the solution of the Dirac equation for an electron (or positron) in presence of an external magnetic field B_j for, say, $j = 3$, leads to the energy eigenvalues,

$$\varepsilon_n = \sqrt{c^2 p_3^2 + m^2 c^4 + 2e\hbar c B n}, \tag{1}$$

where $n = 0, 1, 2...$ are the Landau quantum numbers, p_3 is the momentum component along the magnetic field $\mathbf{B}$ and m is the electron mass. The breaking of the spatial symmetry due to the magnetic field is manifested in the spectrum as an harmonic oscillator-like quantization of the energy in the direction perpendicular to the field. It has the form

$c^2p_\perp^2 = e\hbar cB(2n+1)$. This term combines with the spin contribution $\pm e\hbar cB$, leading to the last term inside the square root in (1). Below we will integrate over p_3 and sum over n and extract the finite contribution to the vacuum energy. The system is degenerate with regard to the coordinates of the orbits center [4].

In the Casimir effect the motion of virtual photons perpendicular to the plates is bounded and we have the photon energy eigenvalues,

$$\varepsilon_s = c\sqrt{p_1^2 + p_2^2 + (\mathcal{P}s)^2}. \tag{2}$$

Due to the breaking of the rotational symmetry in momentum space inside the cavity, only vacuum modes of discrete momentum $p_3 = \mathcal{P}s$ where $s = 0, \pm 1, \pm 2, ...$ are allowed. After taking the sum of all these modes and subtracting the divergent part from it [2], one gets a finite negative term dependent on $\mathcal{P}$, which is the vacuum energy. From this, it was shown by Casimir [1],[2] that a negative pressure appears in between the plates and perpendicular to them.

Starting from the analogy between (2) and (1), which have as a common property a discrete quantization of some of its momentum components, it would be interesting to investigate in parallel both physical systems, especially since a negative pressure arises as due to the zero point electron-positron vacuum energy in an external constant magnetic field.

VACUUM ZERO POINT ENERGIES

The electron-positron zero point vacuum energy in an external electromagnetic field was obtained by Heisenberg and Euler [5]. For the case of a pure magnetic field, the calculation of the zero point energy in the tree level approximation (the virtual particles interact with the external field, but not among themselves) can be made by starting from the spectrum (1). By summing over all degrees of freedom, the resulting term would contain the contribution from the virtual electron-positron pairs created and annihilated spontaneously in vacuum and interacting with the field B. By subtracting a fourth order divergent term, independent of B, and a logarithmically divergent term, proportional to B^2, where both divergences are due to the short wavelength modes, one extracts finally the finite term,

$$\Omega_{0e} = \frac{\alpha B^2}{8\pi^2}\int_0^\infty e^{-B_c x/B}\left[\frac{cothx}{x} - \frac{1}{x^2} - \frac{1}{3}\right]\frac{dx}{x}, \tag{3}$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant and $B_c = m^2c^3/e\hbar = 4.41 \cdot 10^{13}$G is the QED critical magnetic field. (We observe that the addition of a negative infinite term proportional to B^2 absorbs the classical energy term $B^2/8\pi$). As the quantity in squared brackets in (3) is negative, we have $\Omega_{0e} < 0$. (We want to stress at this point that our aim is to study both Casimir and magnetic field problems as independent. Both problems were investigated together to find the magnetic permeability in Casimir vacuum in [6])

The density of energy Ω_{0C} for the Casimir problem may be obtained directly from (2) by summing over s. This is mathematically equivalent to find the thermodynamical potential of radiation at a temperature $T_{Cas} = \hbar c/2d$, according to temperature quantum field theory methods [7]. One gets a finite term, dependent on d, and a four dimensional divergence independent of d. The finite energy density is,

$$\Omega_{0C} = -\frac{\pi^2\hbar c}{720 d^4} = -\frac{c\mathcal{P}^4}{720\pi^2\hbar^3}. \tag{4}$$

Returning to (3), as fields currently achieved in laboratories are very small if compared with the critical field B_c, in the limit $B << B_c$ one can write,

$$\Omega_{0e} \approx -\frac{\alpha B^4}{360\pi^2 B_c^2} = -\frac{\pi^2\hbar c}{5760 b^4}, \tag{5}$$

where the characteristic parameter is $b(B) = \pi\lambda_L^2/\lambda_C$. Here λ_L is the magnetic wavelength defined previously and λ_C is the Compton wavelength $\lambda_C = \hbar/mc$. The energy density is then a function of the field dependent parameter $b(B)$. The expression for (5) looks similar to (4)

Both Ω_{0e} and Ω_{0C} are relativistic negative energies, which suggests that these vacua bears negative masses, which would lead to repulsive gravity with ordinary matter. From these energy densities we will obtain respectively the energy-momentum tensor expression of vacuum in a constant magnetic field, (as the limit of the corresponding expression for matter given i.e. in [8]) and in Casimir effect, and in consequence, the magnetic and Casimir pressures.

THE ENERGY-MOMENTUM TENSOR

For the sake of completeness and correspondence with the quantum gases of charged [8], and neutral particles bearing a magnetic moment [9], we discuss in what follows the vacuum energy-momentum tensor by starting from the quantum relativistic matter tensor, which contains the contribution of vacuum. Usually the calculations are made in Euclidean variables, where x_4 is taken as some "imaginary time", but vectors and tensors will be written here by using covariant and contravariant indices. We will consider the usual QED Lagrangian density L at finite temperature T and with conserved number of fermions $N = Tr\gamma_0 \int d^3x\bar{\psi}(\mathbf{x})\psi(\mathbf{x})$, having associated chemical potential μ. One can write in an arbitrary moving frame the density matrix as

$$\rho = e^{-\beta(u_\nu \mathcal{P}^\nu - \mu u_\nu J^\nu)} \tag{6}$$

where $\mathcal{P}^\nu$ is the momentum four-vector, $J^\nu = Nu^\nu$, and u_ν is the four-velocity of the medium. From ρ, in the rest system, where $\mathcal{L} = \int d^3xL$, one gets the effective partition functional as

$$\mathcal{Z} = C(\beta)\int e^{-\int_0^\beta dx_4 \int d^3x L_{eff}} \mathcal{D}_i A_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi\delta(G)Det\mathcal{P}. \tag{7}$$

Here A_μ is the electromagnetic field, $\bar{\psi}, \psi$ are fermion fields, $C(\beta)$ is a normalization constant. The gauge condition is G and $\mathcal{P}$ is the (trivial) Fadeev-Popov determinant [14], [15],[16]. We have also

$$L_{eff} = L_{(\partial_\nu \rightarrow \partial_\nu - \mu\delta_{\nu 4})} \tag{8}$$

We conclude that the fourth derivative of fermions is shifted in μ: the chemical potential enter into the density matrix through the vector $c_\nu^{(1)} = \mu u_\nu$, and the temperature through $c_\nu^{(2)} = Tu_\nu$. Then, in the rest system, the field operators depend on the coordinate "vectors" $x_\nu = (\mathbf{x}, x_4)$, multiplied by the "momentum" vectors $P_{M\nu} = (\mathbf{p}, p_4)$, where p_4 are the Matsubara frequencies, which are $2n\pi T$ for bosons and $(2n+1)\pi T$ for fermions, where $n = 0, \pm1, \pm2...$ Thus, after taking the quantum statistical average through the functional integration, indicated by the symbol $<< .. >>$, we obtain the thermodynamical potential $\Omega = -\beta^{-1}\ln << e^{-\int_0^\beta dx_4 \int d^3x L_{eff}(x_4,\mathbf{x})} >>$. We observe that the statistical average leads to $<< \int dx_4 \int d^3xL_{eff} >> \rightarrow \Omega$.

The energy-momentum tensor of matter plus vacuum will be obtained as a diagonal tensor (no shearing stresses occur in our approximation) whose spatial part contains the pressures and the time component is minus the internal energy density $-U$. The total energy-momentum tensor is obtained after quantum averaging as

$$\mathcal{T}_\nu^\mu = << T_\nu^\mu >> \tag{9}$$

where

$$T_\nu^\mu = \frac{\partial L}{\partial a_{i,\mu}} a_{i,\nu} - \delta_\nu^\mu L \tag{10}$$

according to [12], where the index i denotes the fields (either fermion or vector components). In abscence of an external field and/or a vector breaking the spatial symmetry, the only nonvanishing contributions from the terms $(\partial L/\partial a_{i,\mu})a_{i,\nu}$ in (10) are those for which $\mu = \nu = 4$. By considering the fermion field contribution as example, we have, by integrating over $x_4, \mathbf{x}$ the first term in the right of (10)

$$\int dx_4 \int d^3x \frac{\partial L(\bar{\psi}(x)\psi(x), A_\mu(x))}{\partial \psi_{,4}} \psi_{,4} = \sum_n \int d^3p \bar{\psi}(p_4, \mathbf{p})(ip_4\mu)\psi(p_4, \mathbf{p}) \tag{11}$$

the last expression leads to

$$\sum_n \int d^3p(ip_4 - \mu)\bar{\psi}(p_4, \mathbf{p})\psi(p_4, \mathbf{p}) = \sum_n \int d^3p[ip_4 \frac{\partial L(p_4, \mathbf{p}))}{\partial ip_4} - \mu\frac{\partial L(p_4, \mathbf{p})}{\partial \mu}] = \sum_n \int d^3p[T\frac{\partial L(p_4, \mathbf{p})}{\partial T} - \mu\frac{\partial L(p_4, \mathbf{p})}{\partial \mu}] \tag{12}$$

Performing the functional average of (10) one gets $<< \int dx_4 \int d^3x(\partial L/\partial\psi_{,4})\psi_{,4} >> = T\partial\Omega/\partial T + \mu\partial\Omega/\partial\mu$. Then one gets the thermodynamical expression

$$\mathcal{T}_j^i = -\Omega\delta_j^i; \qquad \mathcal{T}_4^4 = -(TS + \mu N + \Omega) = -U \tag{13}$$

[13], where $\mathcal{T}_{ij} = p\delta_{ij}$ are the (isotropic) pressures, $S = -\partial\Omega/\partial T$ is the entropy density, $N = -\partial\Omega/\partial\mu$ is the density of particles and U the internal energy density.

For black body radiation in equilibrium [13] it is $\mathcal{T}^i_{bj} = -\Omega_b\delta^i_j$, where $i, j = 1, 2, 3$, and $\mathcal{T}^4_{b4} = -U_b = 3\Omega_b = -\pi^2 T^4/15\hbar^3 c^3$.

In the case when the Lagrangian depends on a nonvanishing field derivative, for instance, $a_{\mu,\lambda} \neq 0$, then the first term in the right hand side of (10) is nonzero and the pressures are given by $\mathcal{T}^i_j = (\partial\Omega/\partial a_{i,\lambda})a_{j,\lambda} - \Omega\delta_{ij}$. This happens when there is an external field $a_\mu = A_\mu = B[-x_2, x_1, 0, 0]/2$ describing a constant magnetic field (taken along the 3-rd axis), which generates non-vanishing spatial tensor terms through the electromagnetic field tensor $< A^\nu_{,\mu} - A^\mu, \nu >= \mathcal{F}^\mu_\nu$. This leads to pressure terms of form $\mathcal{T}^i_j = -\Omega\delta^i_j - \mathcal{F}^i_k(\partial\Omega/\partial\mathcal{F}^j_k)$, or

$$\mathcal{T}^1_1 = \mathcal{T}^2_2 = \mathcal{T}_\perp = -\Omega - B\mathcal{M} \tag{14}$$

where $\mathcal{M} = -\partial\Omega/\partial B$, is the magnetization and $i = 1, 2,\ j = 2, 1$ For matter in an external magnetic field, an anisotropy in the pressures occurs [8], [11]. The anisotropy is due to the arising of a negative transverse pressure, generated by an axial "force": the quantum analog of the Lorentz force, arising when the magnetic field acts on charged particles having non-zero spin [9], and leading to a magnetization parallel to **B**.

THE VACUUM LIMIT. CASIMIR PRESSURES

Quantum statistics at temperature T and chemical potential μ leads to quantum field theory in vacuum if the limit limit $T \to 0$, $\mu \to 0$ is taken (see e.g. Fradkin [7]; the functional average $<< .. >>$ becomes the quantum field average). This is because the contribution of observable particles, given by the statistical term $\Omega_s(T, \mu)$ in the expression for the total thermodynamic potential $\Omega = \Omega_s + \Omega_0$, vanishes in that limit. The remaining term, which is the contribution of virtual particles, leads to the zero point energy of vacuum Ω_0. Thus, we can find the total energy-momentum tensor, and take at the end the limit $T \to 0$, $\mu \to 0$ the quantum field average. If in (13) the quantities $< a_{i,\mu} >= 0$, then

$$\mathcal{T}^i_j = -\Omega\delta^i_j \tag{15}$$

is the isotropic pressure and we conclude that *for the isotropic vacuum, if the energy density* $\Omega > 0$, *the pressures would be negative* (and on the opposite, if $\Omega < 0$, $\mathcal{T}_{ij} > 0$). However, this is not the case if there is a breaking of the spatial symmetry (leading to momentum non-conservation; momentum conservation is restored if we include the sources of the symmetry breaking). These are just the case under study, and they are especially interesting since they provide examples in which divergences can be subtracted, leading to finite negative energy terms, as seen before. The energy-momentum tensor for the Casimir effect in vacuum may be obtained, however, directly by following a complete parallelism with the temperature case, since the four-momentum vector breaking the spatial symmetry has a discrete component $p_3 = \mathcal{P}n$, $n = 0, \pm1, \pm2...$ By choosing the Lorentz gauge $\partial_\mu A_\mu = 0$, one can write

$$\mathcal{T}^3_3 =<< \int_0^{d/\pi\hbar} dx_3 \int d^3x'[A_{\mu,3}\partial L/\partial A_{\mu,3} - L] >> \tag{16}$$

Where $d^3x' = dx_1 dx_2 dx_0$. After quantum averaging (only two degrees of freedom are left; Ω_{0C} is obtained in the Appendix, and coincides with [1]), one has the anisotropic pressures,

$$\mathcal{T}^3_3 = P_{C3} = \mathcal{P}\frac{\partial\Omega_{0C}}{\partial\mathcal{P}} - \Omega_{0C} = 3\Omega_{0C} = -\frac{\pi^2\hbar c}{240d^4} < 0 \tag{17}$$

which is the usual Casimir negative pressure and

$$\mathcal{T}^C_\perp = P_{C\perp} = -\Omega_{0C} = \frac{\pi^2\hbar c}{720d^4} > 0 \tag{18}$$

which is a positive pressure acting parallel to the plates in the region inside them. This is a second Casimir force. (This *is not* the so-called lateral Casimir force reported in [17]). The combined action of both forces suggests a flow of QED vacuum out of the cavity inside the plates, as a fluid which is compressed by the attractive force exerted between them.

One must remark that the usual Casimir pressure corresponds to minus the energy of the blackbody radiation at $T = T_{Cas}$, e.g., $P_{C3} = \mathcal{T}^{C3}_3 \to \mathcal{T}^4_{b4} = -U_b$ and the Casimir energy corresponds to minus the blackbody pressure $\mathcal{T}^{C4}_4 = -\Omega_{0C} \to -\Omega_b$, that is, both tensors are similar under exchange of their $\mathcal{T}^3_3, \mathcal{T}^4_4$ components.

THE MAGNETIZED VACUUM

In the magnetic field case, according to ([11], [8]), the diagonal components lead to a positive pressure $\mathcal{T}_3^{0e3} = P_{03} = -\Omega_{0e}$ along the magnetic field B, and to $\mathcal{T}_\perp^{0e} = P_{0\perp} = -\Omega_{0e} - B\mathcal{M}_{0e}$ in the direction perpendicular to the field. Here $\mathcal{M}_{0e} = -\partial\Omega_{0e}/\partial B$ is the vacuum magnetization, which is obtained from (3) as

$$\mathcal{M}_{0e} = -\frac{2\Omega_{0e}}{B} - \frac{\alpha B_c}{8\pi^2}\int_0^\infty e^{-B_c x/B}\left[\frac{cothx}{x} - \frac{1}{x^2} - \frac{1}{3}\right]dx. \tag{19}$$

One can check easily that (19) is a positive quantity. Moreover, it has a non- linear dependence on the field B. Then it may be stated that the quantum vacuum has ferromagnetic properties (and $\partial\mathcal{M}_{0e}/\partial B > 0$), although in our present one-loop approximation we do not consider the spin-spin interaction between virtual particles.

Concerning the transverse pressure $P_{0e\perp} = -\Omega_{0e} - B\mathcal{M}_{0e}$, we get

$$\mathcal{T}_\perp^{0e} = P_{0e\perp} = \Omega_{0e} + \frac{\alpha B_c B}{8\pi^2}\int_0^\infty e^{-B_c x/B}\left[\frac{cothx}{x} - \frac{1}{x^2} - \frac{1}{3}\right]dx. \tag{20}$$

Both terms in (20)are negative, thus, $P_{0e\perp} < 0$, whereas along the field, the pressure $P_{03} = -\Omega_{0e}$ is positive. This leads to magnetostrictive effects for small as well for high fields. This could be tested by placing parallel (non necessarily metallic) plates parallel to B. Such plates would be compressed in the direction perpendicular to B. Thus, QED vacuum in a magnetic field B is compressed perpendicular to it, and as the pressure is positive along B, it is stretched in that direction. Obviously, for non-metallic plates perpendicular to B, the pressures are positive perpendicular to the plates and negative along them. This is reasonable to expect: the virtual electrons and positrons are constrained to bound states in the external field, but flows freely in both directions along the field. That motion of virtual particles can be interpreted as similar to the real electrons and positrons, describing "orbits" having a characteristic radius of order $\lambda = \sqrt{\hbar c/eB}$ in the plane orthogonal to B, but the system is degenerate with regard to the position of the center of the orbit. It must be stressed that the term $B\mathcal{M}_{0e}$ subtracted by $-\Omega_{0e}$ in $P_{0e\perp}$ is the statistical pressure due to the quantum version of the Lorentz force acting on particles (in the present case virtual) bearing a magnetic moment, which leads to $\mathcal{M}_{0e} > 0$ [9]. In the low energy limit $eB << m^2$ we have $P_{0e\perp} \approx 3\Omega_{0e} < 0$. It can be written

$$P_{0e\perp} \approx -\frac{\pi^2\hbar c}{1920 b^4}, \tag{21}$$

For small B fields of order $10 - 10^3$G, $P_{0e\perp}$ is negligible as compared with the usual Casimir pressure. But for larger fields, e.g. for $B \sim 10^5$ G it becomes larger; one may obtain then pressures up to $P_{0e\perp} \sim 10^{-9} dyn\ cm^{-2}$. For a distance between plates $d = 0.1cm$, it gives $P_{0C} \sim 10^{-14} dyn\ cm^{-2}$, (see below) i.e., five orders of magnitude smaller than $P_{0e\perp}$. Our results show that quantum vacuum in a constant magnetic field may exert pressures, either positive or negative, which means *a transfer of momentum from vacuum to real particles or macroscopic bodies* (as well as in Casimir effect, see below). A similar idea is approached from classical grounds in [10].

DISCUSSION

It is easy to check that for the magnetic field case, the expression for the energy-momentum tensor of vacuum is Lorentz-invariant with regard to inertial frames moving parallel to B. It is also easy to check that the Casimir energy-momentum tensor defined by (4), (17) and (18) remains invariant with regard to Lorentz transformations to inertial frames parallel to the plates. The trace $\mathcal{T}_\mu^{0e\mu} = -4\Omega_{0e} - 2B\mathcal{M}_{0e}$ whereas $\mathcal{T}_\mu^{C\mu} = 0$. However, it is of especial interest to consider the trace of the tensor $\mathcal{T}_{\mu\mu}$ in both cases. In the classical isotropic case it is $\mathcal{T}_{\mu\mu} = \rho + 3p$. In the Casimir case one has $\mathcal{T}_{\mu\mu}^C = 2\Omega_C < 0$ and in presence of the magnetic field it is $\mathcal{T}_{\mu\mu}^{0e} = -2\Omega_{0e} - 2\mathcal{M}B < 0$. It is easy to check that in the magnetic field and Casimir cases both the average pressure $\mathcal{T}_{ii}/3$ and the energy density, are negative, since it is $\langle p_{0e}\rangle = -\Omega_{0e} + 2B\mathcal{M}/3 < 0$, but $\langle p_{0e}\rangle/\Omega_{0e} > 1$, and for the Casimir case we obtain similarly $\langle p_{0C}\rangle = \Omega_{0C}/3 < 0$, (and $\langle p_{0C}\rangle/\Omega_{0C} = 1/3$ as in blackbody radiation).

Quantum vacuum energy has been suggested as a possible candidate to dark energy, leading to a repulsive gravity, equivalent to a cosmological constant [18],[19]. The condition $3p+\rho < 0$ is expected to be fulfilled in Einstein equations assuming the energy density $\rho > 0$, and in consequence the average pressure $p < 0$, which means $w = (p/\rho) < -1/3$. (This also might be understood as a consequence of (15) if $\Omega > 0$).

In considering the effect of magnetized vacuum, we shall assume the magnetic lines of force in intergalactic space as describing curves in all directions (there is no preferred direction for the magnetic field B) , or either, we may assume that there are magnetic domains randomly distributed in space, assuming in each of them the magnetic field as constant, so as to give an isotropic spatial average of the energy-momentum tensor. Concerning the energy term, it must contain the contribution of the average density of the matter creating the magnetic field.

We will refer especially to the extreme case of superdense matter where it is magnetized so that the pressure transverse to B is $P_\perp = -\Omega - B\mathcal{M}$. For $-\Omega \leq B\mathcal{M}$, the transverse pressure vanishes or becomes negative, leading to unstable conditions: the gravitational pressure exerted by the body cannot be balanced by matter pressure, the outcome being an anisotropic collapse [8]. For the energy density under these conditions one can write $U \leq N\mu - B\mathcal{M}$. We observe that magnetic fields decrease the energy density, although we expect that $U \geq 0$ in any case, i.e. under stable conditions the (negative) magnetic energy never exceeds in modulus the rest energy. (This happens for a gas of charged vector bosons, whose ground state has a decreasing but non-vanishing effective mass $\sqrt{M^2c^4 - eBc\hbar} > 0$ [21] (for increasing B)). Under that assumption, the total energy density would be $U \leq N\mu - B\mathcal{M} + \Omega_{0e} > 0$. Thus, we expect that even if the average magnetic vacuum pressure taken in large regions of space have a negative sign, the average energy density of magnetic field+sources should be positive. The situation would be similar to the case discussed after (15). By comparing (5) with the estimated density of visible matter (around 10^{-10}erg/cm^3) [22], it would mean a field of 10^5G. For present cosmology this is too large, since the estimates for the intergalactic magnetic fields are in the range $10^{-6} - 10^{-9}$ G [20].

However, the mechanism is interesting since if we assume the existence of some regions of space having a distribution of strongly magnetized matter, it might be possible to have vacuum average negative pressures leading inside these regions to $\rho + 3\langle p\rangle < 0$, with $\rho > 0$. The mechanism might be interesting also in connection to alternative inflationary models.

The main teaching of the results discussed in this letter is that QED vacuum under the action of external fields having axial symmetry, after spatial averaging, provide the way of extracting finite vacuum negative pressure densities and negative average energies.

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APPENDIX

As pointed out, there is a correspondence between the expressions for the energy momentum tensor in Casimir effect and in the black body problem at temperature $T = T_{Cas}$. The Casimir energy (4) can be shown to be equivalent to the thermodynamic potential of black body radiation at temperature T. We start from the generalized propagator $\mathcal{D}(\epsilon) = [p_4^2 + p_1^2 + p_2^2 + p_3^2 + \epsilon]^{-1}$, where ϵ is equivalent to a squared mass term to be taken as zero at the end. For the photon propagator we demand $\mathcal{D}_0 = \mathcal{D}(\epsilon = 0)$. We start from the quantity

$$\Omega_\epsilon = 2(2\pi\hbar)^{-3}T\sum_s \int d^3p \ln \mathcal{D}(\epsilon). \tag{22}$$

By taking the derivative with regard to ϵ one obtain an expression which can be summed over s [7]. Then, after integrating the result over ϵ, and by taking $\epsilon = 0$ afterwards, leads to the expression

$$\Omega = \frac{2T}{(2\pi)^3\hbar^3c^3}\int d^3p[\ln(1 - e^{\omega/T}) + \omega/T] \tag{23}$$

where $\omega = \sqrt{p_1^2 + p_2^2 + p_4^2}$. The logarithmic term can be transformed by integrating by parts, leading to the usual expression for the black body thermodynamical potential, which is $\Omega(T) = -\pi^2T^4/45\hbar^3c^3$ plus a divergent term, independent from T. But if $\partial\Omega_\epsilon/\partial\epsilon$ is first integrated over p_3, and the resulting expression is integrated over ϵ, the final result is exactly the same as the Casimir energy (4) by replacing T by $\hbar c/2d$ in $\Omega(T)$.

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